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Journal of Teacher Education 2009; 60; 364
DOI: 10.1177/0022487109341150

Journal of Teacher Education
Volume 60 Number 4
September/October 2009 364-379

10.1177/0022487109341150
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Conceptualizing the Work of Leading Mathematical Tasks in Professional Development[1]

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Filling the knowledge gap in the limited research on professional development leaders is an urgent issue if teacher learning is to be improved. This research and development project is studying how leaders learn to cultivate mathematically rich professional development environments. The authors adapted two frameworks from classroom-based research—sociomathematical norms and practices for orchestrating productive discussion—to support leaders' understanding of facilitation of mathematics professional development. In this article, the authors describe the use of these frameworks in their work and argue for a third framework—the mathematical knowledge for teaching. Based on the analysis of their work, they believe that mathematics professional development leaders need to cultivate particular sociomathematical norms for teacher explanation and employ practices for orchestrating discussions to achieve the purposeful development of teachers' specialized knowledge of mathematics for teaching.

Keywords: *mathmatics professional development; leader development; mathematical knowledge for teaching; sociomathematical norms*

In a workshop on their new instructional materials, 30 intermediate and middle grade teachers worked in small groups on a staircase task to determine the number of cubes needed to build the 100th staircase if the first staircase was one cube, the second three cubes, the third six cubes, and so on. When they finished, Sue, the facilitator, asked participants to share how they thought about the problem. Elise came to the overhead and showed her table of values. As she did, she said, "I noticed that my table had 1 then 1 + 2, then 1 + 2 + 3, then 1 + 2 + 3 + 4 and I remember from one of my reviews of high school math that to find the total of these I use $n(n + 1)$ divided by 2. But, I am not an algebra teacher so. . . ." Sue asked if anyone had questions. A teacher asked what n stood for and Elise showed how to plug values into the formula to get the total number of cubes. The group applauded and Sue asked if another person might share. Mario said he also got $n(n + 1)$ divided by 2 and illustrated his approach using the model of the staircase to fit a second staircase of the same size on top to make a rectangle. Pointing to the dimensions of the rectangle he said, "n is the number of cubes along the bottom and the $n + 1$ is the number of cubes along the side. I divided by two because my staircase was half of the rectangle." Teachers signaled how much they liked the use of a visual model with applause and a buzz of praises. The facilitator prompted for another way and Christa showed her approach next. She noticed that if n was the number of cubes in the last column, then the total cubes for one staircase was $n + (n – 1) + (n – 2) + (n – 3)$. . . . She said the total for any staircase was the number of cubes in the last column plus the previous total. "But I ran out of time trying to figure out how to write it." The sharing continued and each presenter was

> applauded. Sue continued to ask, "Is there another way?" until no one else volunteered. Sue then said, "Okay, let's look at one more problem from the chapter before breaking for lunch."

This depiction of professional development (PD) illustrates common practices in PD. The facilitator elicits teachers' varying solutions and invites teachers to question one another in a supportive learning environment (Desimone, 2009; Loucks-Horsley et al., 2003). As professional development leaders ourselves, we have often facilitated such discussions. We use this example as a means to begin a conversation that we, as researchers and facilitators of PD, believe is imperative to advance leaders' learning. In the scenario, although the leader provided a rich task and encouraged teachers to share solutions, the mathematical purpose for teacher learning was not made evident, neither was how teachers' mathematical learning might be useful for supporting students. In advancing teachers' mathematical learning, a leader[2] may need to "slow down" teachers' conversation to explicitly engage the group in mathematical ideas. For example, a leader might ask, "What might be gained from 'seeing' the solution in the visual model? Were there multiple ways to see the pattern in the model and what are the implications of seeing the pattern in different ways for representing a solution symbolically?" Leaders can use these kinds of questions to explore how mathematical models can be used to represent different, albeit equivalent, expressions or to recognize alternative expressions that stem from defining the variable differently. For intermediate and middle grade teachers working with students who are learning about variables and characterizing patterns using words and symbols, the discussion relating the model to the symbolic notion would support teachers who remembered a formula to understand the mathematical reasoning and the use of multiple representations in algebraic thinking.

Because facilitation moves in the scenario focused on displaying the different ways in which teachers solved the task and providing an open forum to ask questions, it was not clear what teachers were to learn. Although there may be many worthwhile purposes to pursue based on the methods and solutions teachers developed, part of the work of leading is deciding on a purpose that is mathematically worthwhile and relevant for a particular group of teachers. Although teachers can be enthusiastically engaged when doing and sharing different mathematical solutions, we contend that more can, and should, be made explicit about the purposes for doing mathematics in PD and the mathematical knowledge teachers need to develop that would benefit their work with students. In this article, we use our study of leader learning to contribute to the limited body of research on leaders' skills and understandings necessary to support teachers' mathematical learning.

The Leader's Role in Professional Development

Leaders of professional development are central to providing opportunities for teachers to gain new understandings of subject matter. The United States has supported numerous initiatives and committed billions of dollars to bolster teacher learning (Birman et al., 2007). From evaluations of the National Science Foundation's Local Systemic Change Initiative projects, we know that a high quality leader makes a difference in the effectiveness of supporting teacher learning in PD (Banilower, Boyd, Pasley, & Weiss, 2006). Yet, little attention has been given to what or how PD leaders learn (Ball & Cohen, 1999; Elliott, 2005; Even, Robinson, & Carmeli, 2003). What leaders of PD need to know and be able to do in their practice is underdefined and understudied, so much so that Even's recent international review of the literature on leader practice focused instead on the missing literature (Even, 2008). Compounding the issue that there is limited research on what leaders need to learn to improve teachers' ability to teach mathematics effectively is the fact that all states are required to provide teachers with high quality learning opportunities (Borko, 2004; U.S. Department of Education, 2001). Reaching all teachers in a substantive way taxes the PD infrastructure beyond its present capabilities. As a result, more teachers are being asked to serve as leaders (Lord & Miller, 2000). The research community has lagged behind in providing insights on how to best support these new leaders as they facilitate teacher learning. Filling the knowledge gap in the research on leading PD is an urgent issue if teacher learning is to be improved and adequately addressed (Lord & Miller, 2000).

The purpose of this article is to share what we have learned about supporting leader learning through a series of seminars aimed at developing leaders' knowledge and skills for cultivating mathematically rich learning opportunities for teachers. In particular, we discuss how our frameworks conceptualizing this work have evolved based on the analyses of data collected during the first 2 years of a 5-year project—Researching Mathematics Leader Learning (RMLL).[3] Our research and development work focuses on one aspect of mathematics PD, when teachers are engaged in solving, discussing, and sharing mathematical work. Although mathematics PD may include other activities, we specifically focus on how leaders learn to attend to doing mathematics with teachers because it is a primary time during PD that teachers may be developing deeper understandings of mathematics. To support their learning about cultivating rich teacher learning environments, leaders explored two frameworks: sociomathematical norms (norms for mathematical reasoning) and a set of practices for orchestrating productive mathematical discussions. The staff of RMLL created and facilitated seminars as learning opportunities for leaders, studied what and how leaders learned about facilitation, and investigated how leaders facilitated PD in their schools and districts.

In this article, we share our developing understandings of what is involved in the work of leading PD and how our frameworks and designs for supporting leaders need to change to better prepare leaders. The research questions investigated are as follows:

Research Question 1: How did RMLL frameworks help leaders make sense of the work of facilitation related to mathematical reasoning in PD?

Research Question 2: How did leaders use these frameworks to support the negotiation of mathematical reasoning in PD?

We report here two central findings associated with our research questions. First, leaders responded positively to using the frameworks as tools for learning to lead mathematically rich discussions. Moreover, the leaders' discussions of these frameworks provided insights on the tensions they experienced in working with adult learners. Second, leaders recognized the importance of having a purpose when facilitating mathematical tasks and were challenged to specify and realize the implications of such purposes in PD. Based on these two findings, we offer an expansion and specification of our frameworks for leader practice in ways that link particular sociomathematical norms for explanation and practices for orchestrating discussions to the purposeful development of teachers' specialized knowledge of mathematics (Ball, Thames, & Phelps, 2008). Before discussing the two findings below, we review the frameworks for leader learning we used in the design of RMLL seminars.

Frameworks for Leader Practice

We drew on classroom research to create frameworks for leaders to use to support teachers' mathematical understandings in PD. Because research on PD suggests that teachers' mathematical conversations tend to move away from mathematics to focus on other (albeit pressing) pedagogical issues (Hill & Ball, 2004; Wilson, 2003; Wilson & Berne, 1999), we looked to classroom research for ways that teachers keep mathematics central in discourse. From this research, we were drawn to ways that Cobb and colleagues supported teachers in advancing mathematical learning by attending to the sociomathematical norms of classroom practice. Their framework, distinguishing social from sociomathematical norms, suggested that learning opportunities are guided by patterns of interaction, both explicit and implicit, that establish how a group works with each other and accomplishes mathematics (Yackel & Cobb, 1996). A second framework adapted from classroom research was Stein, Engle, Smith, and Hughes's (2008) set of practices for orchestrating productive mathematical discussions. These practices support leaders' ability to focus conversations on important mathematical ideas. When conceptualizing RMLL seminars, we saw these two frameworks as guiding what kinds of mathematical explanations should be normative in PD (sociomathematical norms) and how leaders might facilitate these discussions (practices for orchestrating discussion). We used these two frameworks in our seminars to develop leaders' understandings and skills for facilitating mathematical work in PD. We elaborate on the frameworks below.

Social and Sociomathematical Norms

In classrooms, sociomathematical norms guide the nature of the mathematical work that gets accomplished (Kazemi & Stipek, 2001; Yackel & Cobb, 1996). Several factors shape the development of these norms including the students, the specific contexts, the mathematical content, and what is valued and defined as competent participation in a mathematics class (Lampert, 2001). How and what students share of their mathematical thinking is negotiated between the teacher and the students, guiding the nature of mathematical discourse in a classroom. One could imagine a classroom with a norm for privatization of mathematical thinking, where students do not share. Other classrooms may have multiple solutions being shared and a teacher who negotiates with students (a) how and what ideas are explained and justified, (b) the nature of questions posed to dig into important mathematical ideas, and (c) what constitutes mathematical connections across solutions. The sociomathematical norms of a classroom may or may not promote students' mathematical learning.

We adapted Yackel and Cobb's work to consider social norms in PD—the general ways in which teachers engage with one another—and sociomathematical norms in PD—the specific ways in which teachers engage in mathematical work. Norms most commonly discussed in PD literature are social or professional norms that involve how teachers work with one another. Productive social norms, such as critical colleagueship (Lord, 1994) or inquiry into practice (Ball & Cohen, 1999; Cochran-Smith & Lytle, 2001), involve teachers engaged in reflective analysis of practice to learn from it (Jaworski, 2007; Putnam & Borko, 2000). Sociomathematical norms in PD are the ways that mathematical work is interactively accomplished between teachers and a leader (Elliott, Lesseig, & Kazemi, in press; Kazemi, Elliott, Hubbard, Carroll, & Mumme, 2007). These are the norms that together guide how teachers and a leader do mathematics in PD. For example, participants' willingness to share solutions with one another in PD constitutes a social norm. The kinds of mathematical ideas that teachers feel obligated to bring out in their explanations and the particular mathematical justifications[4] pursued during these discussions constitute sociomathematical norms in PD.

To understand the implications of the social and sociomathematical norms framework for PD, we return to the opening scenario. In this depiction of PD, several teachers shared solutions to a task suggesting a social norm of making ideas public. What was shared mathematically and the substance of the explanations indicated particular sociomathematical norms. For example, mathematical references were made (used a table, remembered a formula, used the visual model, noticed a pattern of numbers) to determine the solution. This pattern suggests that a sociomathematical norm in this group was that teachers identify the mathematical processes used to arrive at a solution. However, neither the leader nor teachers felt obligated to pursue the mathematical justifications underlying the processes shared. One could imagine that "using a table" might involve a variety of different mathematical ideas such as focusing on input/output to develop a functional relationship or looking at differences down the table to develop a recursive rule. "Noticing the pattern" might have entailed some understanding of defining a variable and identifying the rate of change and constant. Similarly, given incomplete thinking, the teacher and others might have pursued the mathematical reasoning underlying steps taken, or attempted, in using a formula to arrive at a solution. Bringing these mathematical justifications to light would suggest a different set of sociomathematical norms.

We advance that part of the work of facilitating a group doing mathematics is establishing and cultivating what is entailed in mathematical sharing, what mathematical ideas are made available in explanations, how confusion or incomplete thinking might be approached, and how and by whom mathematical questions are posed. It is certain that attending to the social and sociomathematical norms of a group is not a trivial task. The negotiation of norms is mediated by multiple factors. For example, leaders' capacity to cultivate teachers' understanding of mathematics depends on their ability to know what and how to press for mathematical details that can lead to uncovering what is implicit in mathematical discussions. As leaders work with colleagues in PD, tension is likely to arise as they focus on teachers' mathematical understandings and uncover teachers' potential mathematical confusion. Leaders are also likely to grapple with negotiating teachers' social and intellectual status, communicated when teachers position themselves by their grade level or teaching assignment (e.g., sixth-grade teacher, algebra teacher), as they interact in PD (Kazemi et al., 2007). Negotiating the kinds of questions posed and who is allowed to question involves knowing what mathematical ideas are worth pursuing and a collective sense of security to ask questions of one another.

Practices for Orchestrating Productive Mathematical Discussion

The second framework we adapted from the classroom is a set of practices designed to help leaders think more holistically about orchestrating productive mathematical discussions (Stein et al., 2008). Stein and colleagues identified five practices intended to make teachers' work with students more intentional, rather than simply reacting in the moment to issues that often arise when working with student productions in the classroom. With a similar goal of increasing leaders' intentionality, we adapted these practices to PD to engage leaders in (a) anticipating teacher responses to rich mathematical tasks, (b) monitoring teachers' responses to the tasks during the exploration phase, (c) purposefully selecting teacher work to share in whole group discussions, (d) purposefully sequencing the teacher work that will be discussed, and (e) helping the group make mathematical connections between different teachers' work to develop powerful mathematical ideas. Because many of the greatest, and perhaps most complex, opportunities for teachers' learning take place during teachers' sharing of mathematics, our use of the practices provided a tool for leader planning and facilitation decision making so that sharing time in PD could be purposefully enacted.

The Leader Seminar Model

Leaders engaged in learning about the two frameworks during a series of seminars. The authors of the article collaboratively developed the leader seminars, with two of the authors leading the development and facilitation and the remaining authors collecting research data. Seminar activities introduced the two frameworks for leader practices (sociomathematical norms and practices) in conjunction with video cases of a facilitator working with teachers engaged in solving and discussing mathematical ideas (Carroll & Mumme, 2007).

Design Principles

A set of design principles informed the seminar development:

1. Tasks should have mathematical coherence and important mathematical ideas across the seminars.
2. A way of understanding sociomathematical norms is for leaders to explore the roles that questioning and responding to confusion or error play in negotiating what counts as adequate explanations and justifications in PD.
3. The importance of purpose is central in identifying the reason for engaging and sharing approaches to math tasks and is a central driver of facilitation as leaders engage in learning opportunities.
4. Leaders should have opportunities to engage in connecting the work in seminars to their own work of facilitating teacher learning. Tools can help leaders make links between the analysis of sociomathematical norms and leadership practice.
5. A stance of inquiry should be cultivated during the seminars through which leaders consider the affordances and constraints of particular pedagogical moves and recognize that there are no prescriptions for "the right way to facilitate" mathematics professional development.

The design principles were content and process oriented to encompass both what and how leaders might learn about mathematics PD (Seago, 2007). We identified mathematical generalization as a focus in our seminars so we could work coherently across mathematical[5] topics germane to PD of K-12 teachers. To provide such coherence, we chose tasks that focused leaders on using models and representations to justify generalizations. We focused on leaders' views of the nature of mathematical explanations and justifications in PD because of its central role to PD practice when doing mathematics and as a means to focus on sociomathematical norms.

Leaders considered the role of purpose, the third principle, across a variety of activities in the seminar, for example, during discussions or considering a task to use in PD in "connecting to practice." Connecting to practice activities were a chance for leaders to explore tools that they might use in facilitation. Tools used in RMLL seminars included a template for considering leaders' purposes in facilitating conversations and a template to aid planning for PD sessions through the practices for orchestrating productive mathematical discussions. Finally, recognizing that leaders must negotiate sometimes competing elements such as content, learners, and context to create productive learning environments (Ball & Cohen, 1999), RMLL seminars encouraged a stance of inquiry. Enacting this principle meant that we asked leaders to consider multiple perspectives when making sense of facilitator/teacher moves and contributions. This principle was central to how we wanted leaders to engage in learning in RMLL seminars and with their teachers in PD.

Video Case Curriculum

Video cases were used in the leader seminars because they provided vivid images of the complex work of facilitating teachers' discussions of mathematical reasoning. A case-based approach is an opportunity for leaders to unpack teachers' mathematical thinking, leaders' actions, and the interaction between the two. There is a growing body of knowledge on teacher learning in PD using practice-based materials and video cases in particular (Borko, Jacobs, Eiteljorg, & Pittman, 2008; LeFevre, 2004; Masingila & Doerr, 2002; Seago & Goldsmith, 2006; Sherin & Han, 2004; Stockero, 2008). This technology provides participants the opportunity to learn from practice while not being pressured by the in-the-moment decision-making process required while facilitating (Sherin, 2004). The leader video case seminars are built on this premise.

Social and Sociomathematical Norms Framework

We drew on the framework of norms when designing seminar activities, such as video case discussions, where leaders were asked to consider how a group was engaged in mathematics to see what seemed to be acceptable to the group of teachers and facilitator. Over the course of the seminars, leaders were prompted to notice the nature of questioning, the locus and distribution of kinds of mathematical talk, and the treatment of errors and confusion as a way of paying attention to sociomathematical norms (Kazemi & Stipek, 2001). Our focus on the nature of explanations led us to name four aspects of interaction through which one could identify sociomathematical norms: (a) sharing, (b) justifying, (c) responding to confusion and errors, and (d) questioning. Table 1 compares

Table 1
Norms for Explanation That Support Teacher Learning in Professional Development (PD)

	Productive Social Norms	Productive Sociomathematical Norms
Sharing	• teachers and the PD leader listen respectfully to one another • teachers share solutions or strategies • teachers work together to find solutions to problems • multiple solutions may be explored	• sharing has a purpose of extending and deepening mathematical thinking • sharing consists of explanations that emphasize the meaning of mathematical ideas • mathematical connections among solutions, approaches, or representations are explored
Justifying	• teachers describe and give reasons for their thinking	• justifications consist of a mathematical argument • justifications emphasize why and how methods work
Questioning	• both teachers and the PD leader pose questions • questions support multiple voices and ideas	• questions push on deepening understanding of mathematical ideas
Confusion/error	• confusion and error are accepted as part of the learning process • teachers are not put “on the spot” over incorrect answers • PD leader encourages teachers to clarify their explanations	• confusion and error are embraced as opportunities to deepen mathematical understanding—comparing ideas, reconceptualizing problems, exploring contradictions, pursuing alternative strategies

characteristics of each of these aspects to distinguish social from sociomathematical norms.

Leaders used this chart as a tool when they considered how teachers in the video cases engaged in mathematical talk. Two categories, sharing and questioning, allowed leaders to consider the role of common PD facilitator/participant moves and how they support social and sociomathematical norms of a group. The areas of justification and responding to confusion and errors were two areas that we identified in the research literature as potentially rich moments in mathematics PD when groups make headway on mathematical learning. We focused leaders here so that they could begin to understand this potential.

Practices Framework

Leaders’ opportunity to learn about the practices for teacher sharing was initiated by reading an overview on the practices. This was followed by examining video cases to consider potentially when a facilitator might have used the practices in planning and enacting PD. Finally, we asked leaders to use the practices in a series of connecting to practices activities culminating in a collaboratively constructed plan for a PD session. Connecting to practice activities were opportunities for leaders to negotiate understandings of the constructs raised in RMLL as they “practiced” using them in facilitation decision making. For example, leaders selected and sequenced a series of solutions to a task for a hypothetical discussion. Through discussions of connecting to practice activities, leaders were able to consider how sociomathematical norms and practices for teacher sharing frameworks might apply to their own facilitation. The intent of RMLL seminars was to help leaders develop deeper understandings of how PD leaders purposefully use discussions of mathematics to cultivate mathematically rich learning environments for teachers. Sociomathematical norms and practices for teacher sharing framed how we supported leaders learning about mathematically rich environments.

Purposeful Progression of Activities

Leaders participated in 6 days of leader development work across an academic year. Each seminar (a 2-day time period) was a purposeful progression of events. First, leaders worked on mathematical tasks with consideration for both the mathematics in the task and how teachers might approach it. Leaders’ collective mathematical work and discussions of their mathematical methods set the stage for the centerpiece of the seminar, a video case of a mathematics PD leader engaging teachers in the same task. Leaders discussed both what mathematical explanations were shared in the video case and how participants in the video case engaged in sharing explanations. Leaders then engaged in a connecting to practice activity where they reflected on ideas from the case and the frameworks of sociomathematical norms and practices for sharing and engaged in activities designed to apply these ideas to their facilitation of mathematics PD. After each seminar, leaders were encouraged to complete homework activities examining the ideas from the seminars in their own work or in their observations of other leaders. Each seminar would include multiple passes through this progression of doing mathematics, discussion of video, and connecting to practice. The 6 days of seminars included seven video cases and four connecting to practice activities.

Participants and Data Collection

Seminars were held in two geographically distinct sites: NW (n = 24) and SW (n = 12). All leaders were volunteers from preexisting groups charged with leading math PD in various contexts with teachers. The majority of leaders also worked with K-12 students in some capacity during the day. NW leaders were from 10 suburban to rural districts working with teachers across the K-12[6] spectrum. Two thirds of the leaders had 1 to 3 years of facilitation experience and one third had more than 4 years' experience as PD leaders. NW leaders concurrently participated in a larger initiative developing participants' content knowledge and instructional leadership. The SW leaders, from one large urban district, were mostly elementary focused with more limited experience—ranging from just starting to 5 years of experience facilitating mathematics PD. SW leaders were supporting teachers to enact new curricula and pedagogy.

For more in-depth leader perspectives, we identified a set of leaders during our first 2-day seminar. We call these participants our case leaders. Five (three NW, two SW) case leaders were selected to be representative of the larger group of project participants. The case leaders represented a range of (a) experience facilitating (from 1 to 5 years), (b) grade-level teaching, and (c) contexts for leading PD (elementary, K-8, middle school).

Data for this article were drawn from a larger corpus of data collected on all RMLL seminars.[7] RMLL seminars included activities for seven video cases. For the analysis in this article, we focused on leaders' participation and work in two seminar activities (one video case during Seminar 2 and one all-day connecting to practice activity during Seminar 3), our case leaders' pre- and post-questionnaire data, and case leaders' initial and post-seminar interviews. We targeted these two seminar activities because the video case discussions in Seminar 2 marked the time when the social and sociomathematical norms chart (see Table 1) was distributed, and the connecting to practice activity in Seminar 3 was a culminating event during which leaders collaboratively planned a PD session using the practices for teacher sharing and reflected on the ideas central to RMLL, including sociomathematical norms. These two activities were most useful for the analyses in this article because they required all leaders to collectively share ideas on the frameworks, giving us access to their thinking and their potential utility in planning facilitation. Furthermore, by selecting two seminars, we are able to look across time at how the RMLL frameworks were used by leaders to make sense of the work of facilitation (Research Question 1).

To answer our second research question—how leaders used our frameworks to support the negotiation of mathematical reasoning in PD—we also drew on data from the five case leaders, whom we observed and interviewed (pre/post) facilitating two PD sessions. Observations of case leader PD sessions were determined by selecting a session when leaders would be doing mathematics with teachers for a majority of the time allotted for the session. After case leaders were selected, we identified dates when they would offer PD and coordinated observations. Specific data from the two RMLL seminar activities and case leaders are explained more fully below.

Seminar Data

The data from the seminar events analyzed for this article included (a) fieldnotes constructed from videotaped activities and (b) case leaders' seminar work collected as artifacts. To gain a greater understanding of the two RMLL frameworks during seminars, we also analyzed case leaders' (c) pre- and post-seminar questionnaires and (d) two (initial and post-RMLL) seminar interviews. Fieldnotes from the two seminar activities examined here documented all leaders' experiences (not just case leaders). Fieldnotes were constructed by transcribing videotaped seminar events. Each event was documented with two video cameras, which enabled us to capture interactions in both whole group and at least two small groups. Case leaders were often placed in one of the two videotaped small groups to capture their contributions. Fieldnotes focused on recording all leaders' dialogue (case leaders and noncase leaders) in the seminars and their use of seminar handouts.

Seminar artifacts included case leaders' journals from the video case and connecting to practice activity. A reflection question, recorded in the journal, was posed at the end of the video case Seminar 2. This question asked leaders to reflect on the frameworks of sociomathematical norms and practices for teacher sharing. Journals also contained mathematical solutions generated individually and built on collectively and completed handouts during connecting to practice activities. Journals were photocopied for collection purposes.

A pre/post-seminar questionnaire was administered to collect demographic information (pre) and more substantive questions (post) on our seminar frameworks. For example, we asked, "If you were to explain what are sociomathematical norms to a colleague, what might you say?" Questions also focused on our second framework, practices for teacher sharing. Questionnaire data were meant to gather leaders' written reflections on the seminars and were referenced in RMLL seminar interviews.

Initial and post-RMLL seminar case leader interviews were audiotaped and transcribed. The first interview, conducted after Seminar 1, asked leaders to identify central issues in facilitation and typical practices they used in PD. Leaders also identified where, when, and with whom they facilitated PD. The post-seminar interview allowed researchers to probe case leaders' understandings of the two frameworks by posing follow-up questions to the post-seminar questionnaire. Leaders were asked to elaborate on comments they made about sociomathematical norms and practices for sharing. We also asked them to reflect on a video case they had viewed in the last seminar and to talk about particular social and sociomathematical norms they might have noticed.

Case Leader PD Data

Data for case leaders' facilitation of two PD sessions consisted of a pre-observation interview, observation of an approximately 1½-hour PD session, artifact collection, and post-observation interview. Case leaders' PD sessions were videotaped with one camera and fieldnotes were compiled in real time to capture PD events and times. Case leaders' planning documents, math tasks, and other notes were collected and participants' mathematical work was captured digitally. Audiotapes of case leaders' pre- and post-observation interviews were transcribed.

Observation interviews with case leaders focused on leaders sharing their planning and reflections on the session content and participants. We asked questions on goals, planning processes, and more specific, anticipated teacher solutions. Leaders shared how the session observed may, or may not, link to previous and subsequent PD sessions. Questions were posed to gain a sense of teacher participants' backgrounds and the group's previous experiences working together. Post-observation interviews asked leaders to reflect on the session we observed. Leaders reflected on specific events that showed evidence of meeting their goals and surprises they met within the session. We also asked leaders to discuss their reasoning behind specific decision-making moves we observed. For example, if a leader asked a group to share a particular mathematical idea, we would ask why he or she had made that move and its import for the leader.

Analysis

Analysis of the data used for this article progressed in several phases. We examined fieldnote data with the intent of identifying the meaning that participants made of sociomathematical norms and practices for teacher sharing. While examining fieldnotes, we reviewed videos of seminars, systematically identifying points in the seminar when leaders were directly working with the frameworks. Analytic notes were added to fieldnotes followed by the construction of analytic summaries on leaders' sense making of constructs. These summaries consisted of researchers' comments and evidence on (a) how leaders talked (implicitly and explicitly) about the two frameworks, (b) what sense leaders made of the frameworks and what questions they had, (c) when leaders cited evidence or examples from video cases, PD in general, or classrooms, (d) what connections leaders made between facilitation moves and norms and practices, and (e) when leaders qualified or positioned themselves with statements such as "I am not an algebra teacher" or "You're a middle school teacher so . . ." when doing mathematics or in discussions of video cases. Analytic summaries were distributed across the project team for discussion and debate. When disagreements occurred, researchers reviewed evidence listed in summaries, reread fieldnotes to gain insights on the flow of conversation from which evidence was drawn, and re-viewed video to confirm and disconfirm summary conjectures. Conjectures were revised or refined after the team came to agreement on the meaning of leaders' contributions.

At the same time, we analyzed transcripts of case leader interviews. We wrote analytic memos summarizing leaders' sense making of sociomathematical norms and practices for teacher sharing. Researchers compared analytic summaries that captured case leader participation in seminars, case leader memos from interviews, journal entries associated with norms and practices, and responses to the post-questionnaire to identify patterns in what leaders found useful and what they puzzled over. These analyses allowed us to trace how leaders responded to and made sense of sociomathematical norms and the practices for teacher sharing as well as the challenges they faced in making sense of them.

To analyze case leaders' facilitation, we began by examining the session video, using the fieldnotes outlining the events of the session to note events in the facilitation that evidenced norms, practices for teacher sharing, and researchers' analytic comments on leader/teacher contributions. Next, we reviewed pre- and post-interviews in which case leaders discussed their planning and reflected on their decisions. We identified places where leaders were using the RMLL frameworks implicitly or explicitly, discussed the goals for the PD sessions, or made decisions as they monitored teacher work or orchestrated whole group sharing. We were particularly interested in the way mathematical ideas were treated. We highlighted in interviews when leaders talked about teachers' mathematical ideas, facilitation moves associated with these ideas, and leaders' reflection on the events. We used these data to continue to trace how leaders were making sense of sociomathematical norms and practices for teacher sharing, now in their own practice. These analyses led us to identify common themes across cases and to raise new theoretical questions about how to engage leaders with the idea of sociomathematical norms and practices for teacher sharing.

Results and Discussion

We begin with results that leaders responded positively to both of the frameworks used in the seminars and summarize what about these frameworks helped leaders consider how to support teachers' mathematical understanding. Analysis of seminar and case leader data indicated, however, that the usefulness of the frameworks was tempered by tensions that leaders experienced when working with adult learners. A second finding based on these data suggests that leaders recognized the importance of having a purpose when facilitating mathematical tasks; yet, they were challenged to specify and realize the implications of such purposes in PD. We use data from case leaders to show how leaders used the practices for teacher sharing differently based on their familiarity with the mathematical task and the clarity of their mathematical goals.

Leaders Respond Positively to Sociomathematical Norms and Practices for Orchestrating Discussions

As we reviewed leaders' discussion of the norms framework in fieldnotes from Seminars 2 and 3 and examined case leaders' attempts to use and explain the term *sociomathematical norms* in interviews and questionnaire data, we found that although the terminology of norms was not particularly transparent for them, leaders from both of our sites responded positively to the idea of asking teachers to explain their mathematical reasoning in ways that disentangled mathematically complex ideas and went beyond trying to create a socially polite and respectful climate. During the video case discussion of Seminar 2, after an introduction to norms and the chart on productive norms (see Table 1), leaders' discourse focused on the overall purpose for mathematical explanations in PD, implying it was to develop teachers' understanding of mathematics. Leaders used the descriptive language provided in the chart on productive social and sociomathematical norms to distinguish if video case teachers' and leaders' sharing, questioning, and explanations supported greater understanding. They examined and reflected on leader and teacher contributions in the video case discussion in ways that allowed them to classify the nature of questions, sharing, and teacher confusion to assess if moves resulted in teachers' greater mathematical understanding. One leader group went beyond classifying moves and assessing norms to consider possible alternative questions focused on mathematical issues that might have supported greater understanding. We found it interesting that this group "rehearsed" the questions by phrasing and rephrasing to consider what mathematics a question might elicit and potential ways that it might move video case teachers into productive mathematical territory. Below is an excerpt from the leaders' talk during the video case discussion. In the video case, a teacher asked her peers and the leader in the video for help reconciling how she used her physical model for a task, yet arrived at an incorrect answer (NW Leaders Seminar 2, 2007):

> DI: Okay, so what question could she [the leader] have asked then at this point to move into the justification, how could we have pushed her [the teacher] for deeper mathematical understanding? Would a question [be], "What is your n represent?" Would [that] have been a better direction?
>
> DC: Getting it tied back to the connection to her tower length.
>
> DI: Right, what does *n* have to do now with your square that you kept trying to make?
>
> DC: Or the rectangle that you're looking at, either one, or both.

The leaders thought it was mathematically worthwhile for the video case teacher to make connections across mathematical representations (model and equation) and considered how the video case discussion could have better pursued such connections.

Case leaders suggested that sharing should be a means to support learning. Case leaders' interviews after RMLL seminars and before and after PD observations clearly illustrated leaders' attention to sociomathematical norms and a leader's role in using sharing as an opportunity to further teachers' mathematical understanding. Leaders structured sessions so that particular mathematical ideas would arise for teacher discussion with the intent that these discussions would support deeper understanding. For example, one NW case leader set up a task so teachers' initial conjectures about the sums of consecutive addends were assigned to groups to justify and argue. This structuring of the session allowed the leader and teachers to explore mathematical properties of numbers as they constructed justifications and negotiated the nature of evidence that constituted a sufficient mathematical justification (NW Case CA Observation, PD1, 2007).

SW case leaders suggested very similar notions about how sharing was meant to develop teachers' mathematical learning. One of the case leaders stated in her post-seminar interview, "Your job is to increase your understanding and others' understanding. It is not just about a show and tell. It is about being an interactive participant" (SW Case Leader Interview, 2007). Furthermore, she saw that "norms really emphasize and embed the importance of the mathematical thinking" (SW Case Leader Interview, 2007). All five case leaders, as representative of the larger group, reported that focusing on the productive sociomathematical norms for explanation (as outlined in Table 1) helped them think more deeply about how to facilitate teacher sharing of mathematical conversations that went beyond "serial sharing" of solutions.

Leaders' ideas were tempered by tensions of working with adult learners on mathematics. A number of leaders, especially our case leaders, suggested that, unlike with students where it was their job to ask questions that uncovered potential confusion as a means to support learning mathematics, they experienced tensions around asking questions about colleagues' mathematical thinking. During discussions of video cases, a number of leaders (NW and SW) stated their own reluctance to ask teachers and make their confusions public. As one leader put it, "I was torn as to whether or not to speak up but I didn't want to put them on the spot if they weren't comfortable sharing out about their struggle through that" (SW Case Leader Interview, 2007).

When leaders saw an opportunity to pursue teachers' mathematical learning, they sometimes hesitated. One of our case leaders made the distinction between facilitating colleagues' and students' learning clear for us when he said, "With kids we can [say], 'Hey, let's approach that.' As adult learners, we work together. That is a fine line of, 'I don't want you to feel discomfort. I don't want you to feel inadequate, but I also want you to learn, too'" (NW Case DI Post Interview, PD2, 2008). Facilitating PD for this leader treaded a "fine line" between working on an equal footing with colleagues and identifying where a colleague might need support—his responsibility as a facilitator. Sometimes, this leader mitigated this tension by self-disclosing his own lack of understanding: "I would question the same. . . . It is like I don't understand what is going on there, and I'm really glad she asked it" (NW Case DI Post-Interview, PD2, 2008). However, the leader also noted times when he moved away from teachers' confusion too quickly and suggested that he needed to open up these mathematical ideas for further examination in PD. Understanding how to navigate the fine line between being a colleague and facilitating learning was central to leaders' sense making of sociomathematical norms.

Another issue that made facilitating PD with colleagues complex was that leaders[8] noticed how teachers position themselves and each other (i.e., I am good at math or I struggle with algebra) to reveal their sense of competence around doing mathematics. Early in our initial reflections on our data, we also recognized leaders' verbal positioning when doing mathematics. Similarly, when we observed leaders facilitating PD, we heard teachers making comments about their participation and colleagues. As a result, we identified in our data (RMLL seminar and case leader PD) how leaders and teachers qualified their contributions to discussions. Listed are the types of statements we documented from participants: (a) I'm not a math person; (b) Those teachers know math because they are the middle school teachers (and we're elementary teachers); (c) She will have an equation because she is the algebra teacher and I am not; (d) He will arrive at an equation quickly because he's really smart; and (e) I don't know math but I'm learning and here's a question that occurs to me. What we recognized in our analysis was that in some cases, the qualifying of contributions opened up opportunities for learning (e.g., when someone played the "I'm not good at math" card, asking what she or he believed to be a naïve question, and got everyone to dig deeply into mathematics). In other cases, it closed down opportunities (e.g., when elementary teachers deferred to middle school teachers and didn't pursue appropriate mathematical ideas).

Some of our leaders (NW and SW) clearly identified the negative effects of teacher positioning in PD. They were most articulate about this when discussing PD focused on algebraic reasoning with mixed grade level teachers (e.g., 6-8 or K-8). A few of our case leaders were experimenting with ways to mitigate the unproductive positioning and status issues they had seen arise among their staff. For example, one leader called for teachers to only share solution strategies and not the final solution so that teachers would not defer to the "algebra teacher" (NW Case CA Post-Interview, PD1, 2007). Another NW leader decided that his group would first examine student work on a similar task before doing mathematics as a group of teachers. The leader believed that this sequencing of sessions provided access for teachers and encouraged his algebra teachers to move away from only highly symbolic solutions for a task

(NW Case DI Post-Interview, PD2, 2008). These examples of leaders attending to status differentials through structuring and sequencing PD sessions and listening to leaders discuss the tension of supporting colleagues have spurred us to consider the role that mathematical knowledge plays in PD. Leaders learned that one way to mitigate positioning was by identifying mathematical ideas worth developing that are equally important for teacher learning regardless of grade level or self-identified strength or weakness.

Leaders see practices for sharing useful to support teacher learning. In our first seminar, leaders were asked to read an overview of the practices for teacher sharing, consider them in a video case, and observe a PD session outside the seminar, to reflect on the potential of this framework to guide facilitation. When leaders began RMLL seminars, their pre-questionnaire data suggested overwhelmingly that they thought sharing in PD was a chance for all teachers to display their solutions. In post-seminar questionnaires, it was striking that the majority of leaders felt that they could exercise agency when sharing solutions by selecting and sequencing particular solutions to advance goals. The practices seemed to resonate with leaders as a means to structure teacher discussion, avoiding the often random sharing of solutions. In the NW, many leaders had used the practices in their classroom teaching, and the practices for teacher sharing were a natural extension for thinking about facilitating PD. Although the SW leaders were new to the ideas, they were no less enthusiastic about the potential of the practices for supporting facilitation. One case leader commented on the introduction of the practices in her post-seminar questionnaire: "I have already found this very useful and plan on using it again and again for all the PD sessions I facilitate. I find that it really helps me to mentally prepare for what I will be presenting" (SW Case Leader Post-Questionnaire). Leaders' comments suggested that the practices provided them with an organization for teacher sharing that allowed them to anticipate how sharing might unfold and prepared them for the complex work of facilitating learning. Leaders' enthusiastic acceptance of the practices, as evidenced in leaders' comments, did not mean that all case leaders easily took up the practices in the same ways in their facilitation.

Leaders used practices differentially depending on familiarity of task. In our interviews, we asked case leaders to explain how they had planned for PD and to share potential solutions they anticipated that teachers might use to solve the mathematics task used in their PD. Leaders indicated that they had tried to anticipate solutions and consider which solutions should be shared. Furthermore, they told us that they found it easiest to anticipate teachers' solutions when they had experienced the task as learners with others. In fact, when using a seminar task for *their* PD session, leaders were quite adept at recalling a variety of solutions that were shared during the seminars, and they considered these as plausible solutions to look for during their own facilitation. For example, two case leaders, during their pre-observation interviews, were able to articulate different teacher solutions (without notes) and to anticipate difficulties that teachers might face with a task. This discussion stood in contrast to interviews with leaders when they were less familiar with the task and, thus, had either fewer potential solutions or a general sense of what teachers might do with the task. In the latter cases, when faced with observing teachers' mathematical productions in PD, leaders shared all teachers' solutions without pursuing any particular mathematical idea.

Leaders Recognize the Importance of Purpose in PD

During pre-facilitation interviews on PD planning, leaders would identify a variety of mathematical purposes that could be enacted during whole group sharing when using a task from a project seminar. Indeed, we saw the practices for teacher sharing help leaders construct plans for the whole group discussion rather then leave the activity to chance. Although leaders suggested in their comments that selecting and sequencing teachers' solutions was appropriate in PD, our observations of case leaders' facilitation showed that, at times, case leaders were challenged to select and sequence solutions and to use their purpose to drive the mathematical connections they made across solutions. A number of leaders lamented on this in their post-observation interviews, suggesting that when faced with making decisions in the moment, they struggled with how to adapt their plan and attend to their purpose as teachers made their mathematical ideas explicit through sharing. One leader stated, "I wish we had had more time for closure, to be able to connect the posters. . . . It is hard for me to stay to an agenda because I tend to start thinking about other things that could also be important" (NW Case KN Post-Interview, PD1, 2007). For this leader, the challenge of developing particular mathematical connections across solutions was countered by the temptation to pursue many different issues at once.

Specifying purpose requires negotiation of multiple factors. Our analysis of leaders facilitating PD sessions revealed that the process of identifying the mathematical goal(s) for a PD session can be quite complex. Leaders tried to balance the desire to dig into important mathematics and the pressure they felt to engage teachers in relevant mathematical work. Leaders wrestled with where to place the focus in discussing tasks with teachers, given the variety of mathematical ideas important for their teachers. For example, one NW leader identified several major mathematical ideas that were at play in the task he posed and how the ideas tied with what he considered to be his teachers' needs. He was trying to unpack the mathematical ideas through the way he sequenced solutions, wanting teachers to see how computation methods could be generalized and how all solutions tied to a model of the situation. Through discussion across the teachers' solutions, he wanted the teachers to be clear that assigning variables to represent different parts of the model could result in different, yet equivalent, expressions. He also noticed that he could raise issues relevant to all of his Grade 6-8 teachers on the use and meaning of mathematical properties as teachers shared the equivalence of various algebraic expressions they had generated for the task. This is an impressive list and perhaps more appropriate in developing a trajectory of mathematical ideas to work on over several sessions rather than one, something that the leader also noted when reflecting on the session.

One factor that case leaders suggested required greater consideration in their negotiation of purpose was an overriding concern that tasks engage *all* teachers in mathematical ideas relevant to their work with students. This was identified in a number of ways. For example, one NW case leader suggested that he didn't want to "waste teachers' time" when working in PD. Another NW case leader suggested that she selected tasks that related to ideas across mathematical domains so that everyone would find something relevant at some point. And both NW and SW case leaders noted that they selected tasks central to (or directly lifted from) teachers' curriculum with an occasional shift when asked by an outside source to use another task. We noticed that the relevance of mathematics in a task for many case leaders was determined by how it aligned with teachers' grade level curriculum. Leaders were committed to pursuing mathematical goals important for teacher learning; however, this was constrained by leaders' need to also identify goals relevant to and aligned with school curricula.

Leaders' purposes were underspecified to guide their moves in PD. An issue that many of our leaders faced was articulating a specific mathematical purpose for a task. In the connecting to practice activity in Seminar 3, leaders (both NW and SW) talked in small groups for extended periods of time about purpose. They were clear that purpose was very important for guiding PD. However, the purposes considered were vaguely stated as processes in which teachers would engage. For example, when considering a staircase model of consecutive counting numbers, leaders suggested that plausible goals might be that teachers "consider the critical elements in the task and use prior knowledge to justify a pattern, derive a rule from a pattern, or describe the pattern and be able to explain why" (SW Leaders Seminar 3, 2007). Some NW and SW leaders within a small group pressed for more specific goals, but these efforts tended to be suppressed as conversations continued and a more specific goal was not reached.

Similarly, a number of case leaders stated vague mathematical purposes that focused on ideas difficult to pursue in teacher learning. For example, one NW leader suggested, "Our goal was for them to—well, looking at the relationships between the height and the volume . . . we wanted them to eventually make the connection to rates" (NW Case CA Post-Interview, PD2, 2008). Another leader articulated that her goal for the sums of consecutive addends was "making connections between different models, multiple representations, and pushing for generalizations" (NW Case KN Pre-Interview, PD2, 2008). Based on our observations, when goals were underspecified, leaders had difficulty knowing what mathematical ideas to follow and press on when teachers presented solutions. Indeed, case leaders acknowledged in their PD interviews that they weren't always clear on what to pursue and they were not always able to see evidence of meeting their goal in a session (NW Case CA/KN Post-Interview, PD2, 2008; SW Case Leader Post-Interview, PD1, 2007).

Implications and Evolving Frameworks for Leader Development Work

Based on our analysis of how case leaders made sense of our frameworks of norms and practices, took up these frameworks in their facilitation, and expressed concerns for working with colleagues, we recognized a need to identify more nuanced and detailed purposes for doing mathematics in PD and to explicitly discuss these purposes with leaders to help them connect the work in RMLL seminars with the understandings they need to teach teachers. To support leaders' learning, we are refining our frameworks for leader practice. With a focus on mathematical purpose in PD, our work will help leaders identify the kinds of explanations that should become normative in PD, use practices for orchestrating discussions to pursue very specific mathematical goals, and select/design tasks that are relevant to colleagues. Reflecting on the use of the two RMLL frameworks, we realized that although we had defined for ourselves how the mathematical work in the leader seminars fit together and we articulated particular mathematical purposes to pursue within each task, we needed to be more clear on how these tasks and the mathematical work of leaders might help them build the mathematical knowledge entailed in teaching. It is here that we have found the work of Ball and her colleagues at the University of Michigan to be instrumental (Ball et al., 2008; Hill, Rowen, & Ball, 2005; see also http://sitemaker.umich.edu/lmt/home).

Ball and colleagues have built a framework for understanding the mathematical knowledge entailed in teachers' work in the classroom. In an article published in this journal, they further define content knowledge (as compared with pedagogical content knowledge) entailed in teaching as Common Content Knowledge (CCK, the mathematical knowledge and skills used by any profession using mathematics) and Specialized Content Knowledge (SCK, "mathematical knowledge and skills *uniquely* needed by teachers in the conduct of their work," p. 34, italics added; Ball et al., 2008). CCK is the knowledge necessary to correctly solve a task. SCK is the disciplinary knowledge entailed in the mathematical work that teachers do.

In the context of our work, we think that SCK may provide an important construct to inform leader development. For example, consider the opening scenario in which teachers were able to draw on their knowledge of mathematics to build a table, use a visual area model, consider recursion, and construct explicit rules to solve the staircase task. By understanding the distinctions between CCK and SCK, a leader might recognize that although teachers were drawing on CCK to solve the task at hand, the discussion did not go far enough to explicate how these ideas could be important to teachers working with students. To develop the mathematical knowledge that teachers need to work with students, a leader would want to unpack how a method worked rather than leaving teachers to focus on solutions. Furthermore, through discussion and questioning, a leader would purposefully display a repertoire of solutions to explicate the mathematical territory of the task. Leaders need to know how to specify purposes for doing mathematics in ways that invoke and develop teachers' SCK and identify tasks and discussion prompts that get teachers into the terrain of SCK. They need to know how to pursue this purpose when orchestrating discussion and support the development of sociomathematical norms in ways that unpack teachers' often highly symbolic or incomplete reasoning.

Because SCK is clearly tied to what teachers do in their classrooms, PD focused on developing SCK is highly relevant for teachers. By understanding how a SCK-oriented purpose for PD is tied to classroom teaching and being able to articulate that understanding to teachers in accessible ways, leaders will be able to begin to address the pressure they felt to assure relevance in their PD. Distinguishing between CCK and SCK is a relatively new idea in the field and not necessarily a part of how practitioners frame the work in PD. With a mathematical purpose that is oriented toward invoking and developing SCK, leaders will need to support teachers' unpacking mathematical procedures and concepts in explanations that typically remain compressed (e.g., highly symbolic methods relying on advanced mathematical language) so that all teachers have the opportunities to examine the reasoning behind methods. These kinds of explanations that move away from strictly CCK to SCK may begin to shift the status differentials among colleagues by redefining what teachers mean when they say that someone is "good at math." Our future work with leaders will highlight how articulating a clear purpose opens up or closes down opportunities to learn both CCK and SCK. Furthermore, our work will take up for serious consideration the relevance of SCK to teachers' work.

Further Directions and Conclusion

Our research on leader learning is contributing to the very limited literature on what leaders need to know to facilitate effective learning opportunities for teachers. As more teachers become leaders facilitating PD for their colleagues, we need the field to grow to understand what is entailed in the complex work of learning to lead. We posit that leading PD requires leaders to understand new frameworks for guiding learning opportunities with colleagues. RMLL draws on three frameworks for advancing leaders' understandings and skills for facilitating mathematics-focused PD: sociomathematical norms, specialized content knowledge, and practices for teacher sharing. The SCK framework provides leaders a sharper focus on the nature of sociomathematical norms productive for teacher learning. The framework of practices for teacher sharing is a means for orchestrating productive discussions and deliberately pursuing SCK-oriented goals.

In future work with leaders, we will explore how task choice and discussion prompts are intimately related to the kinds of knowledge to be developed. We will explore the ways in which tasks are posed so that leaders might attend to teachers' development of SCK. For insights on this issue, we have turned to the work of Ball, Bass, and Suzuka and their colleagues in the Mod4 group (see http://sitemaker.umich.edu/mod4/home#) who are constructing tasks for teacher education and professional development to develop teachers' mathematical knowledge entailed in teaching (Suzuka et al., in press). Their work has led us to examine the mathematical entailments of tasks and the ideas that are important for doing mathematics in PD. For example, consider the staircase task presented in the opening scenario. We would suggest that instead of asking for a solution to the task, a leader might present several solutions that involve visualizing the pattern in a number of ways or use different ways of defining the variable. Discussion prompts and questions would explore how solutions provide insights on the range of possible solutions using multiple representations (e.g., symbols, words, tables, graphs, and the model) and what these solutions seemed to uncover in terms of essential understandings (e.g., how models relate to symbolic representations).

Leaders will need to know how to reframe tasks and prompts in new ways to pursue SCK-oriented purposes. RMLL seminars will engage leaders in a purposeful trajectory of tasks that use and develop SCK. These experiences will lay the groundwork for discussion of the dynamic process of identifying SCK-oriented purposes and tasks. Drawing from the work of Ball and colleagues, we imagine that tasks framed to (a) examine the reasoning behind solutions rather than asking for a solution to the problem, (b) understand the logic behind errors presented in example solutions, or (c) explore how varied solutions can be represented with particular models or tools will serve as initial ways to reframe tasks (Ball et al., 2008).

Because we know that leaders face a number of challenges when facilitating colleagues' learning, we will provide leaders opportunities to co-plan using the three frameworks for leader practice: SCK, sociomathematical norms, and practices for teacher sharing. Leaders will identify purposes as they design or redesign tasks within seminars in a supportive environment where they may seek advice and feedback. A new step in Phase 2 of RMLL will provide all leaders with opportunities to facilitate their plans with a small group of RMLL leaders or with colleagues in leaders' districts (cf. Grossman et al., 2009; Lampert & Graziani, 2009). RMLL will support leaders' developing understandings and practices through strong ties between learning opportunities in seminars and learning gleaned from leaders' facilitating of PD.

Considering elements of the mathematical knowledge needed for teaching—CCK and SCK—has been generative for the revision of RMLL. This work will be highlighted to better support leaders and to guide in our theorizing about what leaders need to know. Future RMLL seminars will support leaders' ability to relate mathematical tasks and prompts used in PD to the particular kinds of explanations and justifications they hope to elicit from teachers. Furthermore, the seminars will help leaders learn to articulate how unpacked explanations are useful for teachers' work in the classroom. We will be working with leaders to help them identify what aspects of mathematics to press on and how to design tasks and mathematical trajectories that articulate these mathematical ideas across PD sessions. The opening scenario provided one view of PD where the mathematical activities of teachers were central. However, our research suggests that this approach to PD is not sufficient to develop teachers' specialized mathematical knowledge needed for teaching. Leaders need opportunities to develop new understandings and skills to ensure that PD advances teachers' understanding of mathematics that is needed in teaching. The research and development work that we are engaged in will help us, as a field, understand the complexity of the knowledge and skills that leaders need to cultivate mathematically rich PD environments where teachers have the opportunities to learn specialized mathematics knowledge and will also extend the field's understandings of the work of leader learning.

Notes

1. This article was accepted for inclusion in Volume 60, Issue 3, Theme Issue: Powerful Professional Development Models and Practices.

2. Throughout this article, we use the term *leader* to refer to the person who facilitates mathematics professional development.

3. The research is supported by a grant from the National Science Foundation (ESI-0554186). Opinions expressed in this report are the authors and do not necessarily reflect the views of the National Science Foundation.

4. We use the term *explanation* to refer to descriptions of methods used to solve mathematical problems. The term *justification* is used to include how and why a solution method is mathematically valid.

5. The mathematical content of the seminars emphasized algebraic reasoning by generalizing from arithmetic, generalizing geometric pattern problems, or generalizing number patterns.

6. Two thirds of the NW leaders led professional development with K-12 teachers. However, of the leaders who worked with secondary teachers (Grades 6-12), most facilitated groups at the middle level.

7. Researching Mathematics Leader Learning constructed and used a survey with all leaders using the Learning Mathematics for Teaching items. These data were not considered in this article.

8. NW leaders were more vocal than SW leaders about issues of status. NW leaders suggested that they were aware of status because of their participation in a K-12 mathematics initiative where they

solved mathematics tasks in K-12 groups. SW leaders also identified how teachers might assign competence but they did not identify issues of status.

References

Ball, D. L., & Cohen, D. K. (1999). Developing practice, developing practitioners: Toward a practice-based theory of professional education. In L. Darling-Hammond & G. Sykes (Eds.), *Teaching as the learning profession: Handbook of policy and practice* (pp. 3-32). San Francisco: Jossey-Bass.

Ball, D. L., Thames, M. H., & Phelps, G. (2008). Content knowledge for teaching: What makes it special? *Journal of Teacher Education*, *59*, 389-407.

Banilower, E. R., Boyd, S. E., Pasley, J. D., & Weiss, I. R. (2006). *Lessons from a decade of mathematics and science reform: A capstone report for the Local Systemic Change Through Teacher Enhancement initiative*. Chapel Hill, NC: Horizon Research.

Birman, B., Le Floch, K. C., Klekotka, A., Ludwig, M., Taylor, J., Walters, K., et al. (2007). State and local implementation of the No Child Left Behind Act Vol. 2. Teacher quality under NCLB: Interim report. Jessup, MD: ED Pubs, U.S. Department of Education, Office of Planning, Evaluation and Policy Development, Policy and Program Studies Service.

Borko, H. (2004). Professional development and teacher learning: Mapping the terrain. *Educational Researcher*, *33*(8), 3-15.

Borko, H., Jacobs, J., Eiteljorg, E., & Pittman, M. E. (2008). Video as a tool for fostering productive discussions in mathematics professional development. *Teaching and Teacher Education*, *24*, 417-436.

Carroll, C., & Mumme, J. (2007). *Learning to lead mathematics professional development*. Thousand Oaks, CA: Corwin Press.

Cochran-Smith, M., & Lytle, S. L. (2001). Beyond certainty: Taking an inquiry stance on practice. In A. Lieberman & L. Miller (Eds.), *Teachers caught in the action: Professional development that matters* (pp. 45-58). New York: Teachers College Press.

Desimone, L. M. (2009). Improving impact studies of teachers' professional development: Toward better conceptualizations and measures. *Educational Researcher*, *38*(3), 181-199.

Elliott, R. L. (2005). *Professional development of professional developers: Using practice-based materials to foster an inquiring stance*. Paper presented at the annual meeting of the American Educational Research Association, Montreal, Canada.

Elliott, R., Lesseig, K., & Kazemi, E. (in press). Sociomathematical norms in professional development: Examining leaders' use of justification and its implications for practice. In L. Knott (Ed.), *The Montana Mathematics Enthusiast Journal monograph #7: The role of mathematics discourse in producing leaders of discourse*. Charlotte, NC: Information Age.

Even, R. (2008). Facing the challenge of educating educators to work with practicing mathematics teachers. In T. Wood, B. Jaworski, K. Krainer, P. Sullivan, & T. Tirosh (Eds.), *The international handbook of mathematics teacher education: The mathematics teacher educator as a developing professional* (Vol. 4; pp. 57-74). Rotterdam, Netherlands: Sense.

Even, R., Robinson, N., & Carmeli, M. (2003). The work of providers of professional development for teachers of mathematics: Two case studies of experienced practitioners. *International Journal of Science and Mathematics Education*, *1*(2), 227-249.

Grossman, P., Compton, C., Igra, D., Ronfeldt, M., Shahan, E., & Williamson, P. (2009). Teaching practice: A cross-professional perspective. *Teachers College Record*, *111*(9). Retreived on 01/04/09 from http://www.tcrecord.org/content.asp?contentid=15018

Hill, H. C., & Ball, D. L. (2004). Learning mathematics for teaching: Results from California's mathematics professional development institutes. *Journal for Research in Mathematics Education*, *35*(5), 330-351.

Hill, H., Rowen, B., & Ball, D. L. (2005). Effects of teachers' mathematical knowledge for teaching on student achievement. *American Educational Research Journal*, *20*(2), 371-406.

Jaworski, B. (2007). Theory and practice in mathematics teaching development: Critical inquiry as a mode of learning in teaching. *Journal of Mathematics Teacher Education*, *9*(2), 187-211.

Kazemi, E., Elliott, R., Hubbard, A., Carroll, C., & Mumme, J. (2007). Doing mathematics in professional development: Theorizing teacher learning with and through sociomathematical norms. In T. de Silva Lamberg & L. R. Wiest (Eds.), *Proceedings of the twenty ninth annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education* (pp. 796-803). Lake Tahoe: University of Nevada, Reno.

Kazemi, E., & Stipek, D. (2001). Promoting conceptual thinking in four upper-elementary mathematics classrooms. *Elementary School Journal*, *102*(1), 59-80.

Lampert, M. (2001). *Teaching problems and the problems of teaching*. New Haven, CT: Yale University Press.

Lampert, M., & Graziani, F. (2009). Instructional activities as a tool for teachers' and teacher educators' learning in and for ambitious practice. *Elementary School Journal*, *109*, 491-509.

LeFevre, D. M. (2004). Designing for teacher learning: Video-based curriculum design. In J. Brophy (Ed.), *Using video in teacher education: Advances in research on teaching* (Vol. 10, pp. 235-258). London, UK: Elsevier, Ltd.

Lord, B. (1994). Teachers' professional development: Critical colleagueship and the role of professional communities. In N. Cobb (Ed.), *The future of education: Perspectives on national standards in America* (pp. 175-204). New York: College Board Publications.

Lord, B., & Miller, B. (2000, March). *Teacher leadership: An appealing and inescapable force in school reform?* Newton, MA: Educational Development Center.

Loucks-Horsley, S., Love, N., Stiles, K. E., Mundry, S., & Hewson, P. W. (2003). *Designing professional development for teachers of science and mathematics*. Thousand Oaks, CA: Corwin Press.

Masingila, J. O., & Doerr, H. M. (2002). Understanding pre-service teachers' emerging practices through their analyses of a multimedia case study of practice. *Journal of Mathematics Teacher Education*, *5*(3), 235-263.

Putnam, R., & Borko, H. (2000). What do new views of knowledge and thinking have to say about research on teacher learning? *Educational Researcher*, *29*(1), 4-15.

Seago, N. (2007). Fidelity and adaptation of PD materials: Can they co-exist? *National Council of Supervisors of Mathematics Journal*, *9*(2), 16-25.

Seago, N., & Goldsmith, L. (2006, July 16-21). Learning mathematics for teaching. In J. Novotna, H. Moraova, M. Kratka, & N. Stehlikova (Eds.), *Proceedings of the conference of the International Group for the Psychology of Mathematics Education* (30th) Volume 5, Prague, Czech Republic. (pp. 73-80)

Sherin, M. G. (2004). New perspectives on the role of video in teacher education. In J. Brophy (Ed.), *Using video in teacher education* (pp. 1-27). New York: Elsevier.

Sherin, M. G., & Han, S. Y. (2004). Teacher learning in the context of a video club. *Teaching and Teacher Education*, *20*(2), 163-183.

Stein, M. K., Engle, R. A., Smith, P. S., & Hughes, E. K. (2008). Orchestrating productive mathematical discussions: Five practices for helping teachers move beyond show and tell. *Mathematics Teaching and Learning*, *10*(4), 313-340.

Stockero, S. L. (2008). Differences in preservice mathematics teachers' reflective abilities attributable to use of a video case curriculum. *Journal of Technology and Teacher Education*, *16*(4), 483-509.

Suzuka, K., Sleep, L., Ball, D. L., Bass, H., Lewis, J. M., & Thames, M. H. (in press). Designing and using tasks to teach mathematical knowledge for teaching. In D. Mewborn & H. S. Lee (Eds.), *Inquiry into mathematics teacher education: Monograph of the Association of Mathematics Teacher Education*. San Diego, CA: Association of Mathematics Teacher Education.

U.S. Department of Education. (2001). *No Child Left Behind Act*. Retrieved June 24, 2008, from http://www.ed.gov/policy/elsec/leg/esea02/index.html

Wilson, S. M. (2003). *California dreaming*. New Haven, CT: Yale University Press.

Wilson, S. M., & Berne, J. (1999). Teacher learning and the acquisition of professional knowledge: An examination of research on contemporary professional development. In A. Iran-Nejad & P. D. Pearson (Eds.), *Review of research in education* (pp. 173-209). Washington, DC: American Educational Research Association.

Yackel, E., & Cobb, P. (1996). Sociomathematical norms, argumentation, and autonomy in mathematics. *Journal for Research in Mathematics Education*, *27*, 458-477.

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